PG – 650

III Semester M.Sc. Degree Examination, December 2016
(RNS) (Y2K11 Scheme) (Repeaters)
MATHEMATICS
M 301 : Topology – II

Time : 3 Hours   Max. Marks : 80

Instructions : *i) Answer **any five** questions choosing atleast **two** questions from **each** Part.*
*ii) **All** questions carry **equal** marks.*

PART – A

1. a) Show that if $A \subseteq (y, \tau^*) \subseteq (X, \tau)$ then A is τ^* compact iff A is τ-compact. **4**

   b) Define compact space. Prove that if (x, τ) is compact iff every family of closed sets having finite intersection property has a non-empty intersection. **8**

   c) Prove that if every countable open cover of (X, τ) has a finite subcover then X is countably compact. **4**

2. a) Prove that every second axiom space is a first axiom space and hence show that converse is false. **6**

   b) Prove that Lindeloff property is topological. **4**

   c) Show that a compact metric space is totally bounded. Is the converse false ? Justify your answer. **6**

3. a) Show that a mapping $f : Z \to X \times Y$ is continuous iff Π_x of and Π_y of are continuous. **5**

   b) Show that $X \times Y$ is first countable iff X and Y are first countable. **4**

   c) Prove that if A is closed in (X, τ) and B is closed in (Y, τ) then $A \times B$ is closed in the product topology and conversely. **7**

4. a) Prove that an infinite set with the co-finite topology is a T_1-space. **4**

   b) Define T_2-space. Show that T_1-space does not implies T_2-space. **5**

   c) Show that a compact subset of a Hausdorff space is closed. **7**

PART – B

5. a) Define a T_3-space. Prove that every metric space is a T_3-space. **8**

b) Prove that T_3-space is both topological and hereditary. **8**

6. a) Define a normal space. Prove that a metric space is normal and hence T_4-space. **8**

b) Prove that a compact Hausdorff space is normal. **8**

7. a) Prove that a completely regular space is a regular space. **4**

b) State and prove Urysohn's lemma. **12**

8. a) Show that the space (X, τ) is completely normal iff every subspace is normal. **8**

b) Prove that a regular Lindeloff space is normal. **8**